End Semester Exam

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Sub :- Discrete Structure
Course :- B.Sc. CS (Hons.) 2nd Sem

Answer NO - 3

Symmetric Relation : A relation R on a set A is called symmetric if $(b,a) \in R$ holds when $(a,b) \in R$, i.e., The relation $R = \{(4,5), (5,4), (6,5)$ $(5,6)\}$ on set $A = \{4,5,6\}$ is a symmetric.

Anti-symmetric Relation : A relation R on a set A is called anti-symmetric if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$, is called anti symmetric, i.e. The relation $R = \{(a,b) \to R \mid a \leq b\}$ is anti-symmetric since $a \leq b$ and $b \leq a$ implies $a = b$.

Ans NO - 15

to prove. $1^2 + 3^2 + 5^2 + \cdots (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$
by math. induction.

let check for $p(1) = (2\times 1 - 1)^2 = \frac{1(2-1)(2+1)}{3}$

$= 1 = 1$

∴ it is true
now since it is true for $p(1)$ let this be true for $p(a)$.
then $p(a) = 1^2 + 3^2 + 5^2 + \cdots \cancel{2} + \cdots + 2(a-1)^2 = \frac{a(2a-1)(2a+1)}{3}$

now we have to that it is also true for $p(a+1)$.

$$P(a+1) = 1^2 + 3^2 + \cdots + (2a-1)^2 + [2(a+1)-1]^2$$

$$= P(a) + (2a+2-1)^2$$

$$= P(a) + (2a+1)^2$$

$$= \frac{a(2a-1)(2a+1)}{3} + (2a+1)^2$$

$$= \frac{a(2a-1)(2a+1) + 3(2a+1)^2}{3}$$

$$= \frac{(2a+1)[a(2a-1) + 3(2a+1)]}{3}$$

$$= \frac{(2a+1)[2a^2 - a + 6a + 3]}{3}$$

$$= \frac{(2a+1)[2a^2 + 5a + 3]}{3}$$

$$= \frac{(2a+1)[2a^2 + 2a + 3a + 3]}{3}$$

$$= \frac{(2a+1)[2a(a+1) + 3(a+1)]}{3}$$

$$= \frac{(2a+1)(2a+3)(a+1)}{3}$$

$$= \frac{(a+1)[2(a+1)+1][2(a+1)-1]}{3}$$

$$= P(a+1)$$

$\therefore$ $P(a+1)$ is also true proved.

Answer No - 4

Injection - an injective function is called injection. An injection may also called a one-to-one function;

let $f: x \to y$ be a function. then f is Injective if distinct element of x are mapped to distinct element of y.
that is if x_1 and x_2 are in X such that $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$. also $x_1 = x_2$, $f(x_1) = f(x_2)$

Surjection: let $f: x \to y$ be a function. then f is surjective. if every element of y is the image of that at least one element of X, that is, image $(f) = y$.

Symbolically,

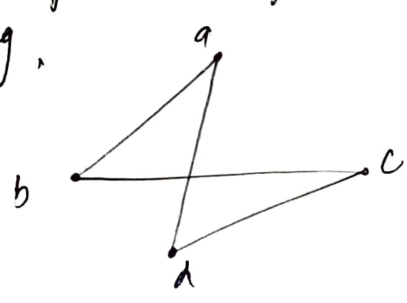
$$\forall y \in Y, \exists x \in X, \text{ such that } f(x) = y.$$

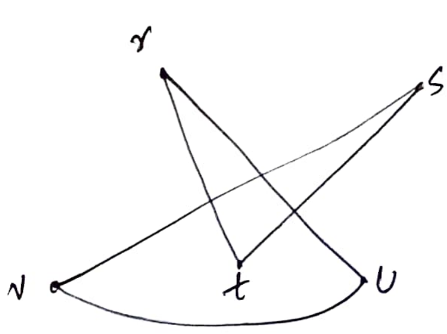
A synonym of surjective is "onto."

Ans No - 1

complete graph:- A simple graph with 'n' vertices is called a complete graph and it is denoted by 'Kn'. In the graph a vertex should have edges with all other vertices. then it will be called complete graph.

eg.





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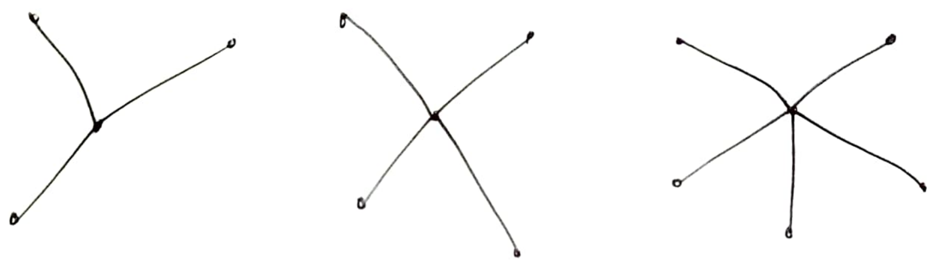
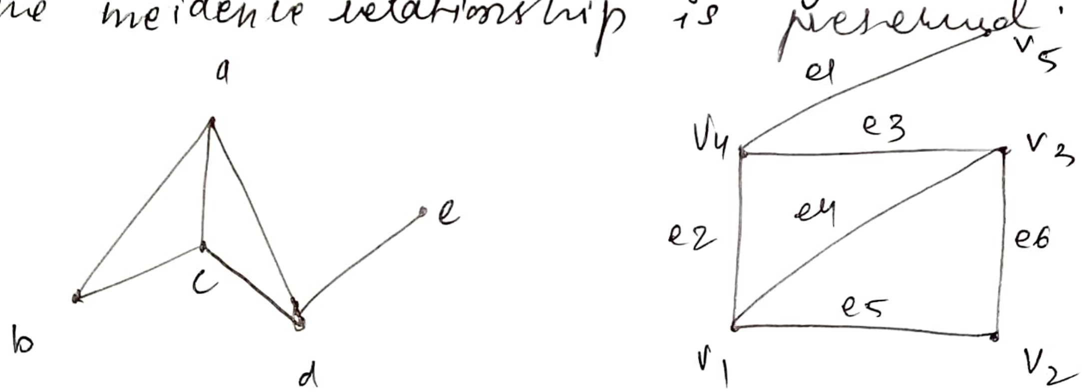
(2) Biparite graph – A complete biparite graph is a graph where vertices can be partitioned into two subset V_1 and V_2 such that the edge of has both end point in same subset, and every possible edge that could connect vertices in different subset is part of the graph

eg.



(3) Isomorphic – two graphs G and G' is said to be isomorphic if there is a onto one correspondence between their edges such that the incidence relationship is preserved.



Answer No - 2

according to question first and second digit are fixed

1st digit – 9

2nd digit – 1

for 3rd digit, we have 8 choices (3, 2, 4, 6, 7, 8, 5, 0)

for 4th digit, we have 7 choices (because repetation is not allowed.

similarly, we have 6 for 5th digit.
and 5 for 6th digit.

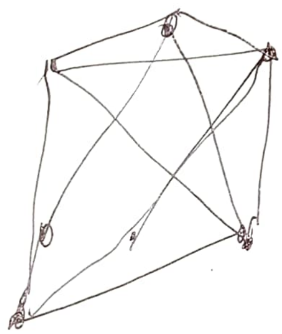
here we will use fundamental principle of multiplication.

$\therefore \quad 1 * 1 * 8 * 7 * 6 * 5 = 1680$

$\therefore$ there are 1680 such possible numbers.

Answer No - 5

the sum of the degrees of the vertices is
$6 * 10 = 60$.
The hand shaking theorem says $2m = 60$
$\therefore$ the number of edges is $m = 30$.



Answer No - 6

given eqn - $T(n) = T(n/2) + n$
by substitution method.

$$T(n/2) = T(n/2^2) + n/2$$

now again.

$$T(n/2^2) = T(n/2^3) + \frac{n}{2} + n$$

now $T(n) = T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} + \frac{n}{2} + n$

$$= T\left(\frac{n}{2^k}\right) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} \cdots\cdots + \frac{n}{2} + n$$

now $\frac{n}{2^k} = 1$

$$n = 2^k$$

taking log

$$\log_2 n = k$$

$$T(n) = T(1) + n\left[\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} \cdots\cdots + 1\right]$$

$$= 1 + n[1+1]$$

$\Rightarrow$ $T n = 1 + 2n$

therefore $T(n)$ is having complexity $O(n)$.

Ans NO - 16

Solve $T(n) = 9T(n-3) + 1$ if $n \geq 1$

given eqn. comparing this from.

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

we get $a = 9$, $b = 3$, $f(n) = n$.

now $n^{\log_b a} = n^{\log_3 9} = n^{\log_3 3^2} = n^2$

$\Rightarrow$ $f(n) < n^{\log_b a}$

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⑦

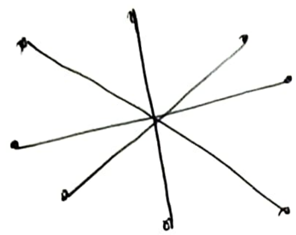
$\Rightarrow \quad T(n) = O(n^{\log_b a})$

$\Rightarrow \quad F(n) = O(n^2)$.

complexity is $O(n^2)$.

Ans NO - 11

only -



only this graph is bipartite, because a complete bipartite graph is partitioned into two subset v_1 and v_2 such that the edge has both end same point in same subset, and every possible group is in connected with ~~for~~ vertices in different subsets. Therefore condition is satisfied.

Ans NO - 7

(1). truth table for

$(p \vee q) \rightarrow (q \wedge \neg p)$

p	q	p ∨ q	¬q	q ∧ ¬p	(p ∨ q) → (q ∧ ¬p)
T	T	T	F	F	F
T	F	T	F	F	F
F	T	T	T	T	T
F	F	F	T	F	T

(2) Truth table for $(p \vee q) \oplus p$.

p	q	$p \vee q$	$(p \vee q) \oplus p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

Answer No - 14

a). total digits available 1, 2, 4, 5, 6, 7, 9
= 7

possible ways of combination without repetition is.

7	6	5	4	3	2	1

ways →

∴ $7 * 6 * 5 * 4 * 3 * 2 * 1 = 5040$

there are 5040 combination

b). first three digit of code are even

3	2	1	4	3	2	1

(2,4,6) (4,6)

therefore no. of ways for getting code is,

$3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1 = 144$

there are 144 combination with first 3 even digit

Ans No - 8

a) $\sim(\sim p \wedge q) \vee q \iff p \vee q$

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$	$\sim(\sim p \wedge q) \vee q$
T	T	T	T	F	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	F	T	T

$p \vee q$	$\sim(\sim p \wedge q) \vee q \iff p \vee q$
T	T
T	T
T	T
F	F

b) $p \wedge (q \vee r) \iff p \wedge (q \wedge r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$q \wedge r$
T	T	T	T	T	T
T	F	T	T	T	F
F	T	F	T	F	F
F	F	F	F	F	F

$p \wedge (q \wedge r)$	$p \wedge (q \vee r) \iff p \wedge (q \wedge r)$
T	T
F	F
F	T
F	T

Aman Yadav

Answer No. 12

given recurrence relation

$f(1) = 1$

$f(n) = n * f(n-1)$ for $n >= 2$

now let put $n = n-1$ for substitution method

$$f(n-1) = (n-1) * f((n-1)-1)$$

$$= n-1 * f(n-2)$$

$$f(n-2) = (n-2) * f((n-2)-1)$$

$$= (n-2) * f(n-3)$$

now

$$F(n) = n * (n-1) * (n-2) * (n-3)$$

$(n-k)$

$$= n * (n-1) * (n-2) \ldots 2 * 1$$

$$\Rightarrow f(n) = n * n\left(1 - \frac{1}{n}\right) * n\left(1 - \frac{2}{n}\right)$$

$$= n \times n\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)$$

$$= n^2\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)$$

here highest degree is n^2 which effects upon all

$\therefore$ complexity of rr is $O(n^2)$

Ans NO - ~~13~~

there are total 80+ students

there are 12 months in yea

~~$\therefore \frac{80+}{12} = 7+$ therefore there~~

$\therefore \frac{80+}{12} = 7+$ therefore there are atleast

7+ students where birth month occur in same month because if we get probability there are 7 students approx every month.

hence proved.